Toward dynamical understanding of the diquarks, pentaquarks and dibaryons

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Abstract. QCD instantons are known to produce deeply bound diquarks. We study whether it may be used as building blocks in the formation of multiquark states, in particular pentaquarks and dibaryons. A simple model is presented in which the lowest scalar diquark (and possibly the tensor one) can be treated as an independent "body", with the same color and (approximately) the same mass as a constituent (anti)quark. In it a new symmetry exists between states with the same number of "bodies" but different number of quarks appear, in particular the 3-"body" pentaquarks can be naturally related to some excited baryons. The limitations of this model are seen from the fact that it leads to light dibaryon H. Another reported work is based on a calculation of a large set of correlation functions for nonlocal operators with 4 to 6 light quarks in the Random Instanton Liquid Model (RILM). The effective interaction between diquarks is found to posses a strongly *repulsive core*, due to the Pauli principle for quark zero modes.

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1. Introduction: instantons and diquarks

The issue of $\bar{q}q$ and qq basic interaction is at the core of hadronic physics. We know that it by no means is restricted to simple one-gluon exchange plus confining linear potential, so familiar from heavy quarkonia. For light quarks the instanton-induced interaction [1] plays a very important or even dominant role, see [2] for review. Just to set a scale, recall that the splitting between a pion and η' coming from it: it is of about 800 MeV, the largest splitting in hadronic spectroscopy. An issue most important for this talk (also discussed in [2]) is existence of deeply bound scalar diquarks, used for building the color superconducting phases at high density [3].

This talk is however about "few-body" problems, in which the number of light quarks (+antiquarks) involved is 4,5, and 6. The issue is especially puzzling at the moment, in view of sharp contrast between the apparent absence of a light dibaryons (see e.g. [6] and also lattice works such as [5] and others), in sharp contrast to surprisingly light pentaquark candidates[1]. Let me on the onset underline two general dilemmas one has to face while discussing the best strategy for building such hadronic states: (i) whether to follow large N_c (number of colors) ideology and to focus on pseudoscalar mesons as clusters, or the "small N_c ideology" and focus on diquarks. (ii) the same in a different words: to use the shell-model ideology, so successful for atoms and nuclei, or to start with the "pairing" first;

[1] References/discussion of current experimental status of this issue is well covered by many specialized talks at this meeting.

Let me also emphasize the main point of the talk: there is a qualitative difference between perturbative and instanton-based forces. One-gluon exchange generates the same interaction between two quarks, whatever other quarks do. Quite differently, an instanton can serve one quark (per flavor) at at time only, due to Pauli principle for 't Hooft zero modes. Thus instantons significantly contribute to clustering (both qq and $\bar{q}q$) at small quark densities only, but are much less able to do so for high density environment. As will be shown below, this creates quite significant repulsive interactions between diquarks, reminiscent of the nuclear core, and rather heavy multiquark states.

The general theoretical reason for the lightness of the scalar-isoscalar diquark state (see e.g.[3]) was known before, it follows from Pauli-Gursey symmetry of the 2-color QCD. In this theory (the "small N_c limit" of QCD) the scalar diquarks are actually *massless Goldstone bosons*. For general N_c, the instanton (gluon-exchange) in qq is $1/(N_c-1)$ down relative to $\bar{q}q$. So the real world with $N_c = 3$ is half-way between $N_c = 2$ with a relative weight of 1, and $N_c = \infty$ with relative weight 0. Loosely speaking, the scalar-isoscalar diquarks are *half Goldstone bosons* with a binding energy of about *half* of that for pion, or about one constituent quark mass.

The binding estimate came from a study of 3-quark correlators a decade ago, in instanton liquid [8] and on the lattice [9]. A marked difference between the nucleon (octet) and Δ (decuplet) correlators at small times has been observed, with the former about a product of that for quark and a very deeply bound *scalar-isoscalar diquark*. The pseudoscalar channel with $\Gamma = 1$ was found to be very strongly repulsive, the vector and axial vector channels are weakly repulsive, with a mass of the order of 950 MeV, above twice the constituent quark mass of the model, $2\Sigma = 840\,\mathrm{MeV}$. The only two channels with attraction and significant binding are: **i.** the *scalar* with $m_S \approx \Sigma$ and $\Gamma = \gamma_5$; **ii.** the *tensor* with $m_T \approx 570\,\mathrm{MeV}$ and $\Gamma = \sigma_{\mu\nu}$ (denoted below by a subscript T).The scalar is odd under spin exchange while the tensor is even under spin exchange. Fermi statistics forces their flavor to be different. The scalar is flavor antisymmetric $\bar{3}$ while the tensor is flavor symmetric 6.

2. A schematic model for pentaquarks based on diquarks

Because of similar mass and quantum numbers, the diquarks may be considered on equal footing with constituent quarks. Certain approximate symmetries then appear [11], relating states with different number of quarks but the same numbers of "bodies". This simple idea is depicted pictorially in Fig.1(left). The $\bar{q}q$ mesons (a) are a well known example of the 2-body objects, as well as the quark-diquark states (b) (the octet diquark-quark baryons). The diquark-antidiquark states (c) are in this model the 2-body objects. In *zeroth* order, the usual non-strange mesons (like ρ, ω), the octet baryons (like the nucleon), and the 4-quark mesons (like $a_0(980)$) are degenerate, with a mass $M \approx 2\Sigma = 840\, MeV$. To *first* order, which includes color-related interactions, the one-gluon-exchange Coulomb and confinement, the degeneracy should still hold, as the color charges and the masses of quarks and diquarks are the same. Only in *second* order, when the spin-spin and other residual forces are included, they split. Note that this new symmetry between N, ρ and $a_0(980)$ is more accurate than the old SU(6) symmetry, which (in zeroth order) predicts $M_N \approx M_\Delta$.

Pentaquarks and dibaryons are in this model treated as 3-body objects, with two correlated diquarks plus an antiquark, are thus related to decuplet baryons, see Fig.1 (d-f). For pentaquarks made of two scalar diquarks the flavor representations are $\bar{3}\otimes\bar{3}\otimes\bar{3} = 1\oplus 8\oplus 8\oplus \bar{1}0$. Using the notations with underline for diquarks as contrast to bar for antiquarks where needed, one can readily see how the pentaquarks observed fit onto an antidecuplet, $\Theta^+(1540) = (ud)(ud)\bar{s} = \underline{SS}\bar{s}$ is an analogue of anti-Ω, and is thus the top of the antidecuplet (the conjugate of the decuplet). New exotic $\Xi(1860)$ are $\underline{UU}\bar{u}$ and $\underline{DD}\bar{d}$, providing the two remaining corners of the triangle. They are the analogue of anti-Δ. The remaining 7 members can mix with one octet, as discussed by Jaffe and Wilczek [10]. making together 18 states in flavor representations ($\mathbf{8}\oplus\overline{\mathbf{10}}$). For

ordinary 3 quarks there is the overall Fermi statistics which ties together flavor and spin-space symmetry and works against the remaining $1 \oplus 8$. There is no such argument for pentaquarks. So how are the additional flavor states $1 \oplus 8$ excluded for pentaquarks?

For diquark-diquark-antiquark all there is left is Bose statistics for identical scalars, demanding total symmetry over their interchange, while the color wave function is antisymmetric. So the only solution [10] is to make the spatial wave function antisymmetric by putting one of the diquark into the P-wave state. It means that such pentaquarks should be degenerate with the excited P-wave decuplet baryons. $M_\Theta = 2\Sigma + \Sigma_s + \delta M_{L=1} + V_{residual}$ where the first 2 terms are masses of the diquarks and strange quark, plus an extra contribution for the P-wave, plus whatever *residual* interaction there might be. One finds that the difference between P-wave and S-wave state is $\delta M_{L=1} = \hbar\omega_\lambda \approx 480\, MeV$ and thus $m_\Theta \approx m^*_\Sigma(3/2) + \delta M_{L=1} \approx 1400 + 480 = 1880\,\mathrm{MeV}$, which is well above the "observed" mass of 1540 MeV.

However, using one scalar and one *tensor* diquark one can do without the P-wave penalty, and the schematic mass estimate now reads $m_\Theta \approx m^*_\Sigma(3/2) + \delta M_T \approx 1400 + 150 = 1550\,\mathrm{MeV}$, which is much closer to the experimental value.

Since the tensor diquark has the opposite parity, both possibilities correspond to the same global parity $P = +1$. Also common to both schemes is the fact that the total spin of 4 quarks is 1, so adding the spin of the $\bar{s}$ can lead not only to $s = 1/2^+$ but also to $s = 3/2^+$ states (which are not yet observed).

So, we conclude that if we only look at the *masses*, it appears that it is better to substitute one diquark by its tensor variant, rather than to enforce the P-wave. Such an alternative scheme provides a different set of flavor representations, $\bar{3} \otimes 6 \otimes \bar{3} = 1 \oplus 8 \oplus 8 \oplus 10 \oplus 27$. The largest representation 27 has particles with quantum numbers of Θ^+ and $\Xi(1860)$, and even more exotic triplets such as Ω-like $sssq\bar{q}$ states. The cascades have isospin 3/2, as observed. However Θ^+ is a part of an isotriplet, with Θ^{++} and Θ^0 partners. The former can decay into pK^+, a quite visible mode.

If one goes a step further, to 6-quark states, for example for 3 ud diquarks, the asymmetric color wave function asks for another asymmetry: to do so one can put all 3 diquarks into the P-wave state, with the spatial wave function $\epsilon_{ijk}\partial_i \underline{S} \partial_j \underline{S} \partial_k \underline{S}$ suggested in the second paper of [3]. This will cost $3(\Sigma + \delta M_{L=1}) = 2700\, MeV$, well in agreement with the magnitude of the repulsive nucleon-nucleon core. However if one considers the quantum numbers of the famous H dibaryon, one can also make those out of diquarks such as $\underline{SDU}$. The resulting wave function is overall flavor antisymmetric with all diquarks in S-states. Thus there is no need for P-wave or tensor diquarks for the H dibaryon. Our schematic model would then lead to a light H never seen. This shows that additive schematic models ignore inter-diquark interaction: the issue we will discuss in the next section.

3. Diquark interaction via the correlation functions

In [7] we attempted to attack the multiquark dynamics directly, propagating 4-6 quarks in the instanton liquid model. (Recall that the number of quarks involved in the model space included in such calculations is equal to the number of instanton zero modes, equal to the number of instantons plus antiinstantons, which is typically 200-300.)

We start with the operators for the two diquark systems[2] of all possible types, with a variable splitting between two diquark operators at each time. For spatially antisymmetric states the probability to find both diquarks at the same point must vanish by Bose symmetry[3]. For each of the four possible cases we look at the dependence of the correlators on the number of flavors

[2] Of course these systems carry color and should be complemented by a heavy antiquark or another diquark, which is factored out.

[3] In most lattice works, for example, the operators used so far are only local, which prevents from approaching all P-wave states.

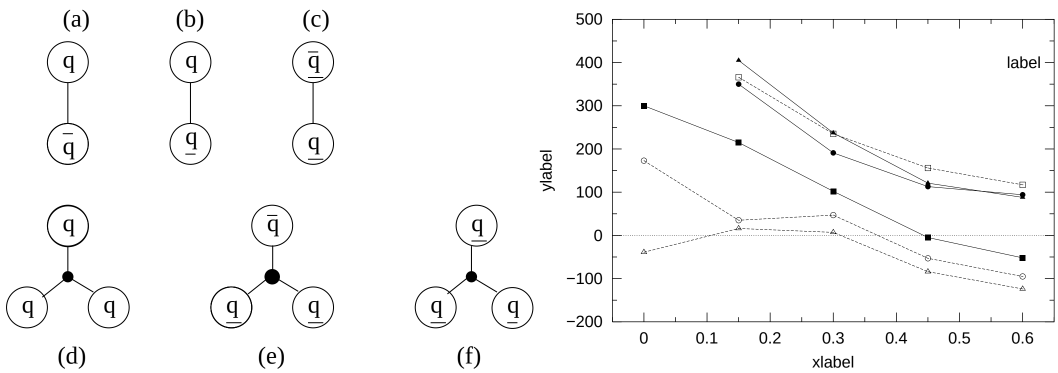

Figure 1. (left) Schematic structure of (a) ordinary mesons, (b) quark-diquark or octet baryons, (c) diquark-antidiquark states or tetraquarks, (d) decuplet baryons, (e) pentaquarks and (f) dibaryons.
(right) Effective interaction potential $V(d)$ for the two diquark systems $\mathbf{3}_C\mathbf{3}_F\mathbf{S}$ $N_f = 3, 4$ $(blacksquare)$; $\mathbf{3}_C\overline{\mathbf{6}}_F\mathbf{A}$ $N_f = 2, 3$ (•), $N_f = 4$ $(blacktriangle)$; $\overline{\mathbf{6}}_C\mathbf{3}_F\mathbf{A}$ $N_f = 3, 4$ $(square)$; $\overline{\mathbf{6}}_C\overline{\mathbf{6}}_F\mathbf{S}$ $N_f = 2, 3$ $(circle)$, $N_f = 4$ $(triangle)$. The uncertainty in $V(d)$ is about 50 MeV.

N_f, which translates into the number of exchange diagrams. At $N_f = 4$ all four quarks are different and there are no exchange diagrams, at $N_f = 3$ there is one, and at $N_f = 2$ we have two of them. Accordingly we would expect their contribution to grow for decreasing N_f.

There are two ways to use the correlators: The usual one is go to the largest τ (Euclidean time) possible and fit a mass from the logarithmic slope (very difficult). We use another one also, namely fit the effective interaction at small τ for 2,3 diquarks placed at a variable distance. Some examples are shown in Fig.1(right). One can clearly see a *repulsive core* at small distances d of the diquarks, which gets stronger at smaller number of quark flavors. The repulsion reaches roughly 300 MeV and the width of the core corresponds to approximately the instanton radius $\rho = 0.35$ fm.

No quite light states other than meson+baryon were found so far for pentaquarks, and all operators with with 3 scalar diquarks lead to flavor singlet H with a large mass ~ 3 GeV. Much more detailed studies are to follow as the work is in progress.

3.1. Acknowledgments

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